A few remarks on the analysis of energy transfer through any periodic current and voltage waveforms

Marek T.Hartman and Mansour Hashad
Gdynia Maritime University/ Department of of Marine Electrical Power Engineering, Gdynia, Poland

Abstract— Numerous publications point out that each of the so far presented proposals how to describe power states has some defects, they all lack the physical interpretation or the universal character. Yet, the application of a given power theory to analyze the electric circuit was not preceded by the proof of the rightness of such a procedure in relation to the character of this circuit. The authors of this paper seek the answer to the question whether a given power theory can be successfully applied in the analysis of both linear and non-linear circuits.
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I. Introduction

One of the fundamental, and at the same time the most controversial, issues in electrical engineering is a universal description of power properties of electrical circuits with non-sinusoidal, periodic characteristics of voltage and current. Research on this issue has been carried out since the nineties of the 20[th] century and is associated with Ch.P. Steinmetz. In 1892 Steinmetz arguing with Henbach [6] presented the measurements results of voltage and current in a circuit with the arc lamp obtained by W.B.Tobey and G.H.Walbridge. [7]. Analyzing the presented results, Steinmetz noticed that in the circuit with the arc lamp, despite the lack of a phase shift between the characteristics of voltage and current, the measured apparent power S is bigger than the measured active power P, that is S > P.

For the next decades different theories known as "power theories" have been presented. Among the authors of these concepts are the outstanding names inseparably connected with electrical engineering: These are Professors: Illovici and Budeanu as well as the Polish scientists - Fryze or Czarnecki - and many others [3,4,5]. Numerous publications point out that each of the so far presented proposals how to describe power states has some defects, they all lack the physical interpretation or the universal character. Nevertheless, all the theories agreed as far as the instantaneous power p(t) and the active power P were concerned. The active power P was defined as the mean periodic value from the instantaneous power. Yet, the application of a given theory to analyze the electric circuit was not preceded by the proof of the rightness of such a procedure in relation to the character of this circuit. The authors of this paper seek the answer to the question whether a given theory can be successfully applied in the analysis of both linear and non-linear circuits.

II. Remarks concerning the calculations of the instantaneous power $p(t) = u(t)i(t)$ in electrical circuits

On the basis of the Standard [2], it can be stated that Budeanu's power theory is considered to be the standard tool to analyze circuits with non-sinusoidal voltage and current characteristics. For non-sinusoidal changeability of the source voltage $u(t) = \sqrt{2}\,U\sin(\omega t)$ supplying the <u>*linear load*</u> (the authors' underlying), the current in the circuit is described by the equation $i(t) = \sqrt{2}\,I\sin(\omega t - \Theta)$, where: U, I are the RMS values of voltage and current respectively. The instantaneous power $p(t)$ is described by the relation

$$p(t) = u(t)i(t) \tag{1}$$
$$p(t) = p_a(t) + p_q(t) \tag{2}$$

where:

$p_a = UI\cos\Theta[1-\cos(2\omega t)] = P[1-\cos(2\omega t)]$; $P = UI\cos\Theta$

$p_q = UI\sin\Theta\sin(2\omega t) = -Q\sin(2\omega t)$; $Q = UI\sin\Theta$

For non-sinusoidal characteristics of voltages and currents, the standard [2] states that the instantaneous power p(t) is described by the equations (1) and (2), where the equation

$$p_a = \sum_h U_h I_h \cos\theta_h [1-\cos(2h\omega t)] \tag{3}$$

contains all the constituents, the mean periodic value of which is not equal to zero and where the equation

$$p_q = \sum_h U_h I_h \sin\theta_h \sin(2h\omega t)] + \sum_{\substack{m\neq n \\ m,n\neq 1}} 2U_m I_m \sin(m\omega t + \alpha_m)\sin(n\omega t + \beta_n) \tag{4}$$

contains the constituents, the mean periodic value of which is equal to zero. The angle $\Theta_h = \beta_h - \alpha_h$ is an angle between the phasors U_h and I_h.

According to the formula (1) the instantaneous power $p(t)$ is the product of the functions $p(t) = u(t)i(t)$, each of which depends only on time. Mathematically, this

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statement can be written down in the following way: $u = f(t)$, $i = g(t)$. There are two possible cases:

A) the voltage is the current function (or vice versa) e.g. on the elements R, L, C and in the real source. In this situation one function, e.g. the voltage is the input and the other, e.g. the current is the output

or,

B) the voltage is not the current function (and vice versa) e.g. on the switches and in the ideal sources. In this "fixatory" situation there is neither input nor output.

Let us consider the electric circuit (fig. 1), in which there appears energy transfer form the "supply source" to the place, where this energy is converted into useful work.

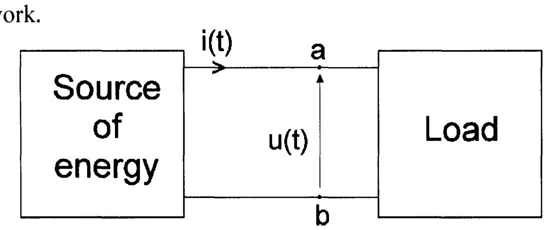

Fig. 1. The electric circuit

In the circuit from figure 1 there appear or there do not appear mutual relations between the current $i(t)$ and the voltage $u(t)$. If these values depend on each other on the basis of physical laws, e.g. Ohm's law, they must be written down as the relations $i(t) = f[u(t)]$ or $u(t) = g[i(t)]$. So, the speed of energy transfer, that is the instantaneous power is described by the following relation

$$p(t) = u(t)\cdot f[u(t)] = i(t)\cdot g[i(t)] \tag{5}$$

In general, *the function p(t) can describe the energy transfer in the linear or non-linear circuit.*

Let us consider the electric circuit as in figure 1, in which $u(t)$, $i(t)$ will be the instantaneous non-sinusoidal characteristics of the voltage and the current of the period T. Assuming that the functions describing non-sinusoidal voltage and current meet Dirichlet's conditions, it is possible to present them with an infinite Fourier series in the forms

$$u(t) = \sum_n \sqrt{2}U_n \cdot \cos(n\omega t + \alpha_n) = \sum_n u_n(t) \tag{6}$$

$$i(t) = \sum_m \sqrt{2}I_m \cdot \cos(m\omega t + \beta_m) = \sum_m i_m(t) \tag{7}$$

So, the relation (5) assumes the form

$$p(t) = u(t)\cdot f[u(t)] = i(t)\cdot g[i(t)] = \\ [\sum_n u_n(t)]\cdot f[\sum_n u_n(t)] = [\sum_m i_m(t)]\cdot g[\sum_m i_m(t)] \tag{8}$$

For the linear circuit the principle of superposition is satisfied and the equation (8) can be written down in the following form

$$p(t) = [\sum_n u_n(t)]\cdot[\sum_n f(u_n)] = [\sum_n i_n(t)]\cdot[\sum_n g(i_n)] = \\ = [\sum_n u_n(t)]\cdot\{\sum_n f[u_n(t)]\} = u(t)\cdot i(t)$$

or as

$$p(t) = u(t)\cdot i(t) = (\sum_n u_n)\cdot(\sum_n i_n) = \\ = u_1(i_1+i_2+..+i_n)+u_2(i_1+i_2+..+i_n)+...u_n(i_1+i_2+..+i_n) \tag{9}$$

In the notation of the equation (9), we made use of the *commutativity law* of both: addition and multiplication. If the circuit is *nonlinear*, then it is not possible to make use of the principle of superposition and the equation (8) remains in the unchanged form. *In that case the function (8) is noncommutative, so first it is necessary to perform the addition of the instantaneous values of voltage and current harmonics, and then to perform multiplication.* It is essential not to confuse the lack of sense of performing the commutativity law of addition and multiplication with a mathematical possibility of its performance. Mathematically, there is always such a possibility. Yet not always does this operation have any physical sense.

III. Remarks on the so far known "power theories"

The authors think that the remarks included in chapter 2 allow presenting some criticism concerning the descriptions of " power theories"

3.1 Theories based on the orthogonal Fourier distribution of voltage and current

The proposals of the descriptions of power states made on the basis of the distribution of voltages and currents harmonics into a Fourier series contain the definition of the active and reactive power as the *sum* of active and reactive powers of all harmonic characteristics $u(t)$ and $i(t)$. A given current harmonic is the output to the input given in the form of the appropriate voltage harmonic, according to the principle of superposition. *Therefore, the widespread and well-known theory of Budeanu can only be made use of (if any) in the analysis of linear circuits.* Applying this method in Standard [2] for the analysis of nonlinear circuits (an example of the calculations of the circuit with a thyristor: Annex A, Theoretical Example, A1. Single-Phase non-sinusoidal) must be regarded as the factual mistake.

3.2 Theories based on the orthogonal distribution of voltage or current – Fryze's theory and its derivatives

Criticizing Budeanu's theory, Czarnecki pointed out in, among others [1], the lack of connection between Budeanu's reactive power Q_B and the oscillation component of the instantaneous power $p(t)$. According to him, the power Q_B does not provide any information about the existence of power supply oscillations between the source and the load. He also questioned the usefulness of Budeanu's theory in designing the compensators improving the power factor.

Demonstrating the interpretation defects of Fryze's power theory [1], Czarnecki analyzed the reactive component of Fryze's current (or Fryze's reactive power). In his opinion Fryze's theory does not create the basis for designing the reactance compensators as well as compensating the loads that generate harmonics [1].

Czarnecki writes about the power theory CPC " *"The main idea of the power theory was to reveal physical phenomena deciding about the value of the power supply current and associate with these phenomena the separate components of the current, that is to assign a physical meaning to the current components. In this sense the CPC theory is the heritage of Fryze's power theory"*(citation from [1], the authors' underlying).

The unquestionable so far – to the knowledge of the authors - value of Fryze's distribution is the active component $i_a(t)$, that is the active current of the resistance load of the conductance equivalent to G_e defined by Fryze as

$$i_a(t) = \frac{P}{\|u\|^2} u(t) \tag{10}$$

The active current $i_a(t)$ is the current of the resistance load, which at the same voltage $u(t)$ has the smallest effective value enabling the dissipation of the same active power *P*. The authors think that such an interpretation of the current $i_a(t)$ is the quantitative (mathematical, boundary, optimization etc.) interpretation and on no account it is the physical interpretation. The authors' doubts will be illustrated by the following examples.

Example 1. Supposing we have an electrical circuit that is presented in figure 2

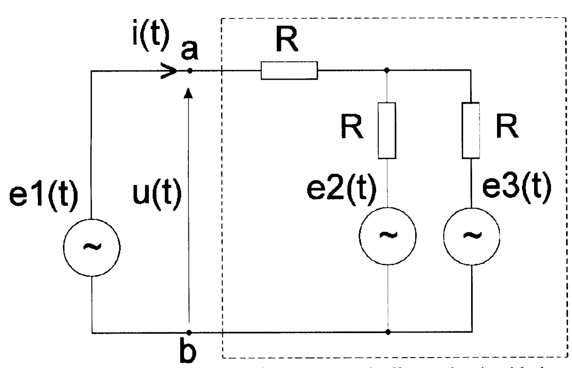

Fig. 2 The scheme of a three-phase symmetric, linear circuit with the balanced load

The circuit from figure 2 is the circuit of the three-phase net (IT type) with the balanced, linear, purely resistance R , in which the current $i(t)$ has the sinusoidal waveform.

Let us write down the voltages in the circuit phases from figure 2 in the following forms

$$e1(t) = \sqrt{2}U_1 \sin(\omega t + \alpha_1) + \sum_{n \in N} \sqrt{2}U_{3n} \sin(3n\omega t + \alpha_{3n}) \tag{11}$$

$$e2(t) = e1(t - \frac{T}{3}), \tag{12}$$

$$e3(t) = e1(t + \frac{T}{3}), \tag{13}$$

then: the voltage $u(t)$ and the current $i(t)$ are

$$u(t) = e1(t) \text{ and } i(t) = \frac{\sqrt{2}U_1}{R} \sin(\omega t + \alpha_1) \tag{14}$$

In the circuit from figure 2 the voltage components, which are the multiple of the third harmonic, cancel each other and in the circuit there appears only the basic harmonic of the current. This situation is well known from the circuits of three-phase nets of the "IT" type. The active power characterizing the energy transfer in the points (a,b) is equal to: $P = \frac{U_1^2}{R} = I_1^2 R$

The phenomena occurring in the circuit from figure 2 are well-known and they do not create any controversy. But, when we make use of the current distribution $i(t)$ according to Fryze's orthogonal components, the active component of the current (equation 10) is:

$$i_a(t) = \frac{P}{E^2} u(t) = \frac{P}{E^2} [u_1(t) + \sum_n u_{3n}(t)] \tag{15}$$

where $E^2 = U_1^2 + \sum_n U_{3n}^2$

It results from the equation (15) that the active component of the current $i_a(t)$ contains the third harmonic and its multiples, which is obviously not true. The current $i(t)$ (equation 14) is the sinusoidal characteristics, so the existence of the reactive components (according to Fryze's theory) is obviously not a true statement and has nothing to do with the physics of phenomena..

Example 2. Supposing we have a nonlinear circuit, in which the voltage and the current in the circuit have the following form

$$u(t) = u_1(t) + u_2(t) + u_3(t) + u_5(t) \tag{16}$$

$$i(t) = i_1(t) + i_2(t) + i_3(t) + i_4(t) \tag{17}$$

where: 1,2,3,4,5 is the order of harmonic of the voltage or the current.

According to the definition of the effective value and Perseval's theorem, we obtain the RMS value of the voltage *U*

$$U = \|u\| = \sqrt{U_1^2 + U_2^2 + U_3^2 + U_5^2} \tag{18}$$

the RMS value of the current *I*

$$I = \|i\| = \sqrt{I_1^2 + I_2^2 + I_3^2 + I_4^2} \tag{19}$$

the active power *P*

$$P = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T u(t)\cdot i(t)dt = \\ = \frac{1}{T}\int_0^T [u_1 i_1 + u_2 i_2 + u_3 i_3 + \ldots + ..]dt = P_1 + P_2 + P_3 \tag{20}$$

It is obvious that the active power P is the sum of the active powers of the harmonics of the order: 1,2 and 3.

On the other hand, while based on decomposition of the current $i(t)$ into Fryze's components, we obtain

$$i_a(t) = \frac{P}{\|u\|^2}u(t) = \frac{P}{\|u\|^2}[u_1(t)+u_2(t)+u_3(t)+u_5(t)] = \\ = i_{a1}(t)+i_{a2}(t)+i_{a3}(t)+i_{a5}(t) \tag{21}$$

and

$$i_{bF} = i(t) - i_a(t) = i_1(t)+i_2(t)+i_3(t)+i_4(t) \\ -[i_{a1}(t)+i_{a2}(t)+i_{a3}(t)+i_{a5}(t)] \tag{22}$$

It results from the equations (21) and (22) that Fryze's theory and its components erroneously assign the physical meaning to the current components because:

- in the active component $i_a(t)$ of Fryze's current there appeared a *harmonic of the fifth order*, which does not exist in the current $i(t)$.

- in the reactive component $i_{bF}(t)$ of Fryze's current there appeared a constituent corresponding to the *harmonic of the fifth order*, which does not exist in the current $i(t)$. *Thus, the current component that is the harmonic of the fifth order is the same harmonic occurring simultaneously as the active component (with the positive sign - plus) and the non-active component (with the negative sign - minus !!!).*

The active power P (equation 21) will be the sum of the powers coming from the harmonics of the order 1,2,3 and 5, which is obviously in contradiction to the conclusion achieved on the basis of the reality (equation 20).

To illustrate the mentioned conclusions we will perform an analysis of the non-linear circuit presented in figure 3. This circuit consists of a series connection of the power supply source of the periodical characteristics $e(t)$, the ideal rectifying diode D and the resistor R. Supposing the figures are respectively:

$$R = 2(\Omega)\,, T = 0.02(s)\,,\ \omega = \frac{2\pi}{T}\,, e(t) = 100\cdot\sin(\omega t) + x(t)$$

where:

$$x(t) = 0 \; if \; (0 < t \le \frac{T}{2})$$

and

$$x(t) = -200 \; if (\frac{T}{2} < t \le T)$$

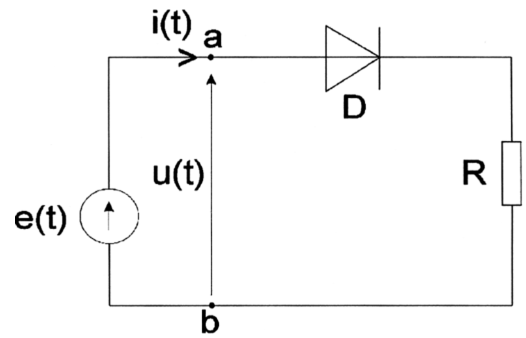

Fig. 3 The nonlinear one-loop circuit

The characteristics of the voltage $u(t)$ in the points a-b is presented in figure 4a, the characteristics of the current $i(t)$ - in figure 4b and the characteristics of the current $i_a(t)$ - in figure 4c.

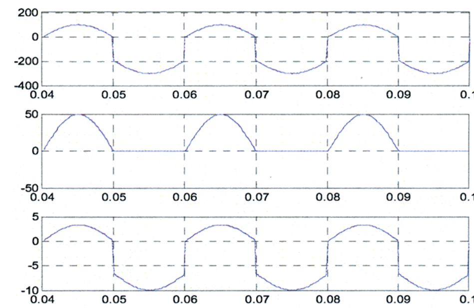

Fig. 4 Characteristics of the voltage $u(t)$, the current $i(t)$ and the active component $i_a(t)$ in the circuit from fig.3.

The values characterizing the power states of the circuit from figure 4 are the following: the active power of the load (resistor R) is P=1250(W), the RMS value of the voltage U=194,2(V) and the RMS value of the current I=25(A).

The distribution of the current $i(t)$ and the component i_a(t) into a Fourier trigonometric series is as follows:

$$i(t) = \frac{50}{\pi} + 25\cdot\sin(\omega t) - \sum_{n=2,4,6,...}\frac{100}{\pi\cdot(n-1)(n+1)}\cos(n\omega t)$$

$$i_a(t) = \frac{P}{U^2}u(t) = 0.033\cdot u(t)$$

where: $u(t) = -100 + 227.3\cdot\sin(\omega t) + \sum_{n=3,5,7,...}\frac{400}{\pi\cdot n}\sin(n\omega t)$

It results from the presented analysis that the current $i(t)$ is the pulse current of the effective value I=25(A) and of the mean value (the constant component) equal to (50/π). The current $i(t)$, apart from the basic harmonic, contains only even harmonics. Yet, in the current $i_a(t)$ there appeared odd harmonics 3,5,7,.. (they appear also in the non-active current with the negative sign), which explicitly shows that *the current $i_a(t)$* (and at the same time the second components of this distribution) *is the creation of purely mathematical nature, which does not have anything to do with the physics of phenomena.* We can make an interesting remark concerning the constant component of the current $i_a(t)$, which is supposed to flow in the reverse current direction of the diode.

The active component of the current $i_a(t)$ from the definition of Fryze's theory is the function proportional to the voltage $u(t)$. In the circuit from figure 3, we assumed that $u(t)=e(t)$. This assumption is only true for an ideal source. In such a case the current $i_a(t)$ is not the function of the reactive (non-active) component $i_{bF}(t)$. In the circuit with the real source the situation is different. The components: $i_a(t)$ and $i_{bF}(t)$ depend on each other, because a change in any of them will cause a change in the voltage $u(t)$. In such a situation theoretically, it is not accepted to treat the components of the current as independent of each other. It means then that there is no possibility of compensating the non-active component without a change of the active power .

Summing up, it can be stated that there exist serious limits to the application of Fryze's theory. Thus, the theories based on the orthogonal decomposition of the voltage/current can not be regarded as the universal ones describing the energy transfer in electrical circuits.

IV. Conclusions

In the authors' opinion, a historical division of voltages and currents into "sinusoidal" and „non-sinusoidal" has proved inappropriate. Irrespectively of the shape of voltage or current characteristics an essential role is played by the character of a circuit and a kind of function describing energy transfer. *The criterion is the acceptability of applying the commutativity law of multiplication and addition of the constituents of the current and/or the voltage. In other words, it is necessary to answer the question: won't a change of any component in a given mathematical distribution/decomposition of the voltage/current (e.g. neutralization by subtracting from both sides of the equality and physically by its compensation) cause a change in the other constituents in the physical circuit ? If the answer is "no", then such a mathematical distribution loses sense (it is not acceptable to apply to it the commutativity property of multiplication and addition of the constituents, because the constituents are independent of each other) and at the same time all other operations and conclusions based on them become pointless.* The descriptions of linear and nonlinear circuits are fundamentally different in this regard and the so far used normalizing standards do not define these differences.

The considerations concerning the function $p(t)$ describing the speed of the power transfer in electrical circuits explicitly indicate that **the power theories making use of Fourier distribution of voltages and currents can be applied only in linear circuits.** Such a distribution results just from the principle of superposition.

The power theories based on the Fryze's orthogonal decomposition of voltage or current (and all its derivative distributions) have numerous defects and limits. The most important limit of these theories is the fact that the decomposition of a given current/voltage depends on voltage/current. Consequently, the commutativity law of multiplication and addition of the distribution constituents is observed only when voltage/current does not depend on current/voltage. In real circuits this case appears only in the circuits, in which the impedance of the voltage source is very small/the admittance of the current source is very big in relation to the power of the load. This is a necessary condition but it is not the sufficient one (compare example 1, example 2). Therefore, the theories based on Fryze's orthogonal decomposition do not describe the real physical phenomena occurring in both: the linear circuit and the nonlinear circuit.

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